Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MFP4

Unit Further Pure 4

Thursday 22 May 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -0.6 & -0.8 \\ 0 & 0.8 & -0.6 \end{bmatrix}$$

represents a rotation.

(a) State the axis of rotation.

[1 mark]

(b) Find the angle of rotation, giving your answer to the nearest degree.

[2 marks]

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 1

2 (a) Factorise the determinant

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

as a product of three linear factors.

[4 marks]

(b) The matrices $\mathbf{A}$ and $\mathbf{B}$ are such that

$$\mathbf{AB} = \begin{bmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{bmatrix} \quad \text{and} \quad \det \mathbf{A} = z^2 - y^2$$

Given that $\det \mathbf{AB} \neq 0$, find and simplify an expression for $\det \mathbf{B}^{-1}$.

[3 marks]

QUESTION PART REFERENCE

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3 The matrix $\mathbf{M} = \begin{bmatrix} 1 & 4 & 2 \\ 3 & k & 3 \\ 2 & k & 1 \end{bmatrix}$, where k is a constant.

(a) Show that $\mathbf{M}$ is non-singular for all values of k.

[3 marks]

(b) Obtain $\mathbf{M}^{-1}$ in terms of k.

[5 marks]

(c) Use $\mathbf{M}^{-1}$ to solve the equations

$$\begin{aligned} x + 4y + 2z &= 25 \\ 3x + ky + 3z &= 3 \\ 2x + ky + z &= 2 \end{aligned}$$

giving your solution in terms of k.

[4 marks]

QUESTION PART REFERENCE

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4 Three vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ are such that $\mathbf{u} \times \mathbf{v} = \mathbf{u} \times \mathbf{w}$, where $\mathbf{u} \neq \mathbf{0}$ and $\mathbf{v} \neq \mathbf{w}$.

Show that $\mathbf{v} - \mathbf{w} = \lambda \mathbf{u}$, where λ is a scalar.

[3 marks]

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5 The points A, B, C and D have coordinates $(1,\ 3,\ p)$, $(4,\ 5,\ 2)$, $(2,\ 1,\ -1)$ and $(6,\ 5,\ 0)$ respectively, where p is a constant.

(a) Write down the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ in terms of p.

[2 marks]

(b) Show that $(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}$ is of the form $m(5 - 2p)$, where m is an integer to be found.

[5 marks]

(c) In the case where $p = 2.5$, describe the configuration of the points A, B, C and D. Justify your answer.

[2 marks]

(d) In the case where the vectors $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ define the edges of a parallelepiped of volume 60, find the possible values of p.

[3 marks]

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6 The plane transformation S is a shear and is represented by the matrix $\begin{bmatrix} a & b \\ c & -2 \end{bmatrix}$, where a, b and c are constants.

(a) Show that $2a + bc = -1$.

[2 marks]

(b) Given further that $(2, 2)$ is an invariant point of S, find the values of a, b and c.

[4 marks]

(c) Show that all lines of the form $y = x + k$, where k is a constant, are invariant lines of S.

[3 marks]

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7 The line l_1 has Cartesian equations

$$\frac{x-1}{4}=\frac{y-2}{-2}=\frac{z+1}{3}$$

and the line l_2 has vector equation

$$\mathbf{r}=\begin{bmatrix}4\\3\\c\end{bmatrix}+\lambda\begin{bmatrix}5\\-1\\0\end{bmatrix}$$

where c is a constant.

The plane Π_1 contains the lines l_1 and l_2.

(a) Show that an equation for the plane Π_1 is $x+5y+2z=d$, where d is an integer to be found.

[4 marks]

(b) Find the value of c.

[1 mark]

(c) The plane Π_2 has equation $2x-y+2z=4$.

(i) Find the acute angle between the planes Π_1 and Π_2, giving your answer to the nearest $0.1°$.

[4 marks]

(ii) Find an equation of the line of intersection of Π_1 and Π_2, giving your answer in the form

$$(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$$

[5 marks]

QUESTION PART REFERENCE

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8 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{bmatrix} p & q \\ q & p \end{bmatrix} \text{ where } p \text{ and } q \text{ are constants and } q \neq 0 .$$

(a) Find the eigenvalues of $\mathbf{M}$ in terms of p and q.

[4 marks]

(b) Find corresponding eigenvectors of $\mathbf{M}$.

[3 marks]

(c) Write down a matrix $\mathbf{U}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{U}\,\mathbf{D}\,\mathbf{U}^{-1}$.

[2 marks]

(d) Show that $\mathbf{M}^n = \mathbf{U}\,\mathbf{D}^n\,\mathbf{U}^{-1}$.

[2 marks]

(e) Given that $p = 0.6$ and $q = 0.4$ and $\mathbf{M}^n \to \mathbf{L}$ as $n \to \infty$, find the matrix $\mathbf{L}$.

[4 marks]

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END OF QUESTIONS